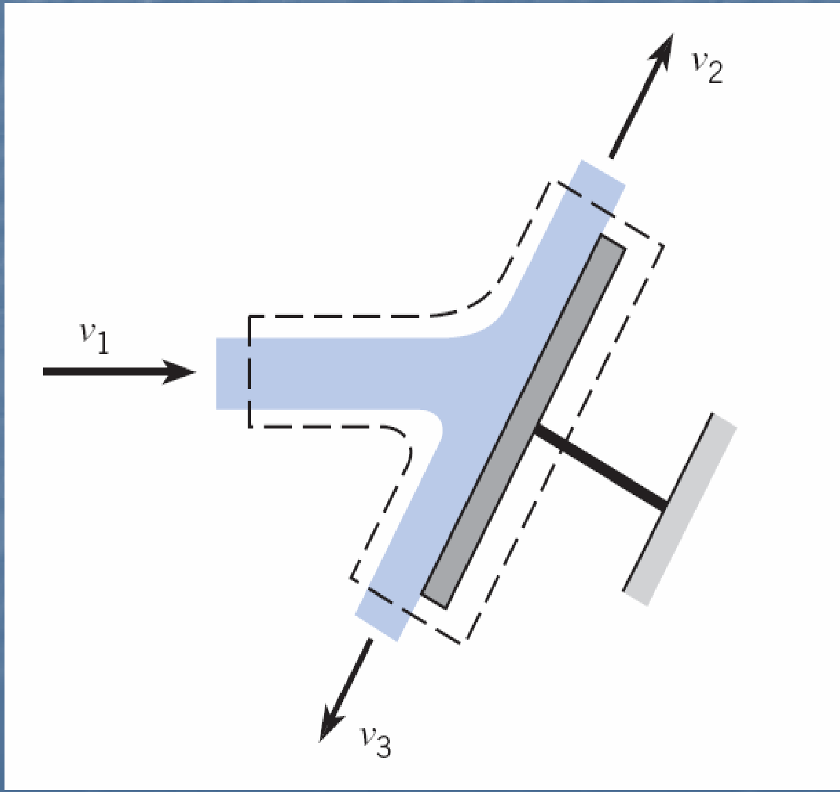
Vanes

A vane is a structural component, typically thin, that is used to turn a fluid jet or be turned by a fluid jet. **(example: a blade in a turbine)**

For a vane or a blade, the following assumptions are considered:

1. Pressure forces are atmospheric.
2. Neglect changes in elevations.
3. Neglect viscous forces



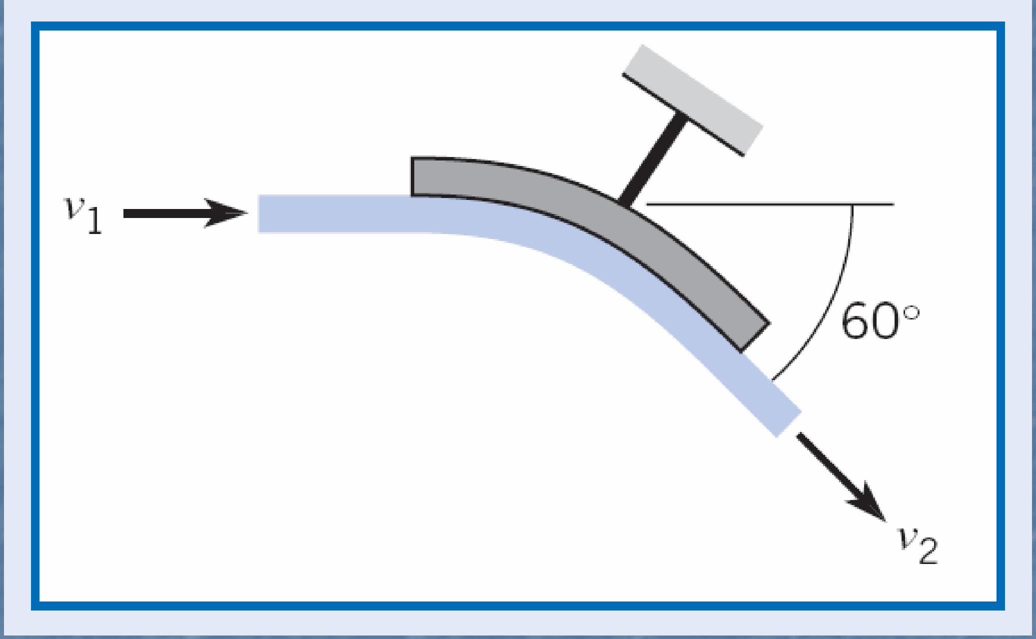
Using Bernoulli's equation

$$\text{Bernoulli's Eqn., } p_1 + \gamma z_1 + \frac{\rho v_1^2}{2} = p_2 + \gamma z_2 + \frac{\rho v_2^2}{2}$$

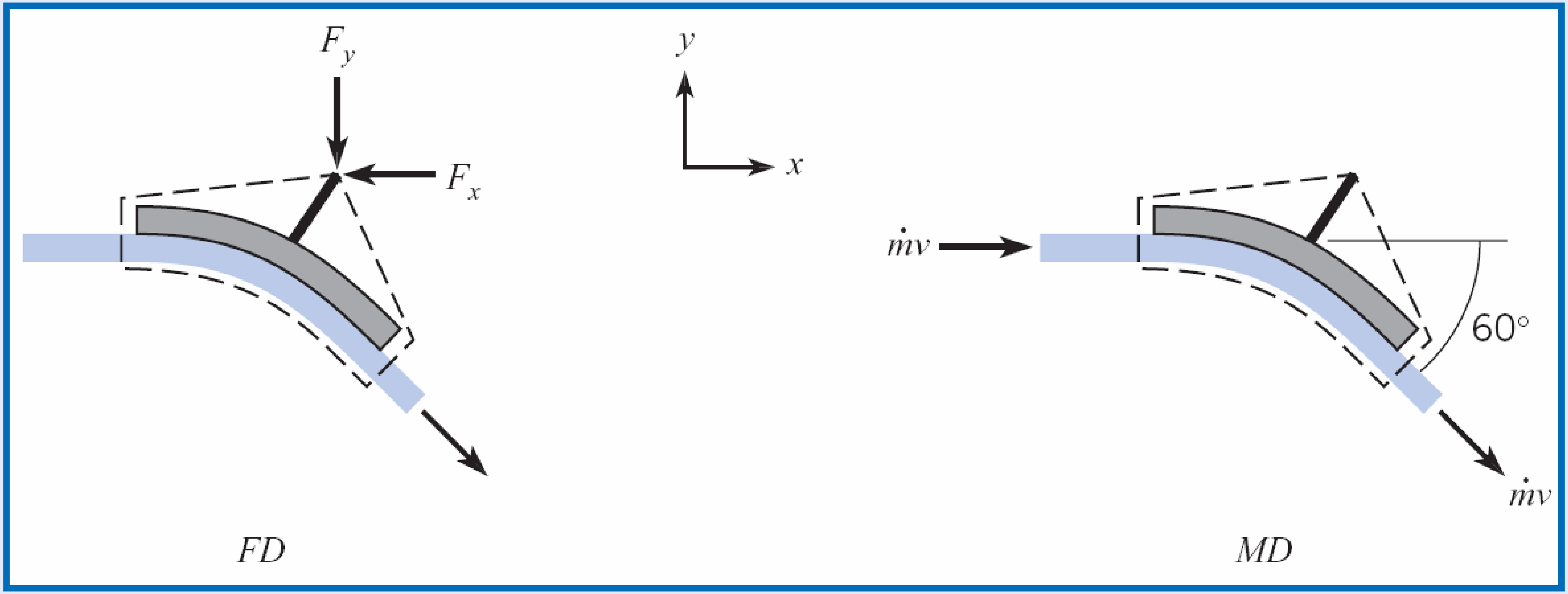
$$V_1 = V_2 = V_3$$

Example (6.4)

A water jet is deflected 60° by a stationary vane as shown in the figure. The incoming jet has a speed of 100 ft/s and a diameter of 1 in. Find the force exerted by the jet on the vane. Neglect the influence of gravity.



Find the force exerted by the jet on the vane?



From the force diagram, $\sum F = -F_x i - F_y j$

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$

From the momentum diagram, $(Note\ that\ v_1 = v_2)$

$$\sum F_x i = \sum_{CS} (\dot{m}v)_{out} - \sum_{CS} (\dot{m}v)_{in} = (\dot{m}v\cos 60)i - (\dot{m}v)i$$

$$\sum_{CS} F_Y j = \sum_{CS} (\dot{m}v)_{outY} - \sum_{CS} (\dot{m}v)_{inY} = -(\dot{m}v \sin 60) j - 0$$

Evaluation of the x direction momentum equation gives

$$F_x = \dot{m}v(1 - \cos 60°) = \rho A v^2 (1 - \cos 60°)$$
$$= (1.94 \text{ slug/ft}^3)(\pi \times 0.0417^2 \text{ ft}^2)(100^2 \text{ ft}^2/\text{s}^2)(1 - \cos 60°)$$
$$= 53.0 \text{ lbf}$$

In the y direction,

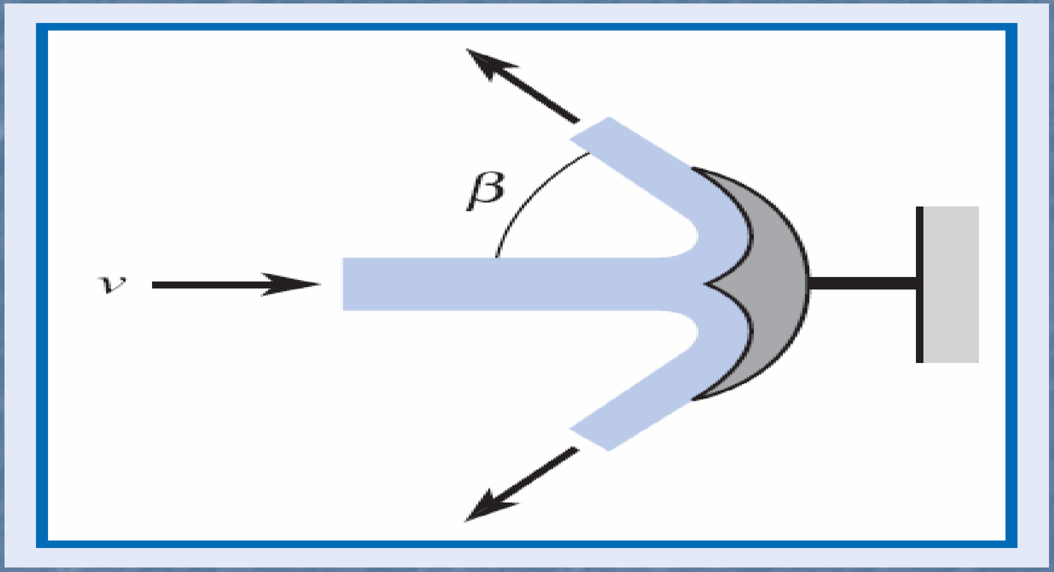
$$F_y = \dot{m}v \sin 60° = \rho A v^2 \sin 60°$$
$$= (1.94 \text{ slug/ft}^3)(\pi \times 0.0417^2 \text{ ft}^2)(100^2 \text{ ft}^2/\text{s}^2) \sin 60°$$
$$= 91.8 \text{ lbf}$$

Note: **positive values of** F_x, F_y **means that the forces of the jet is opposite in direction to the forces required to hold the vane.**

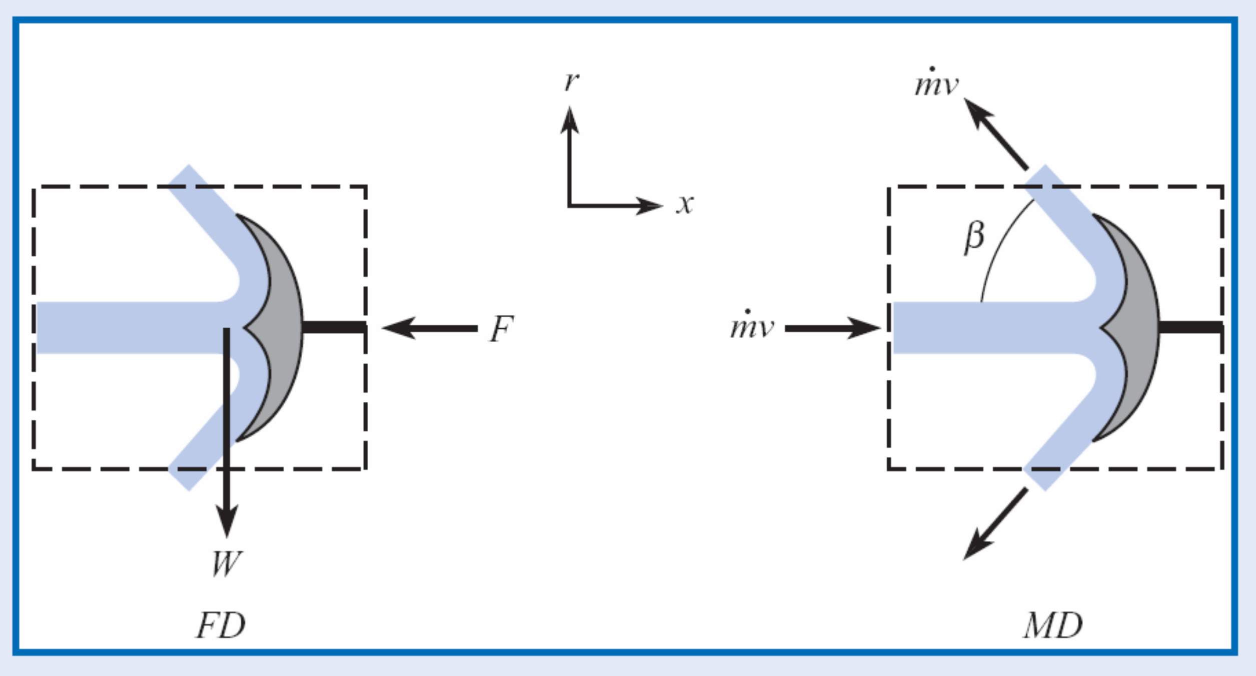
$$F_{jet} = (53.0) i + (91.8) j$$

Example 6.5

As shown in the figure, an incident jet of water with density ρ, speed v, and area A is deflected through an angle β by a stationary, axisymmetric vane. Find the force required to hold the vane stationary. Express your answer using ρ, v, A, and β. Neglect the influence of gravity.



Find the force (F) to hold the vane stationary?



From the force diagram, $\sum F_x = -F$

The momentum accumulation = $\frac{d}{dt}\int_{cv} v_z \rho dQ = 0$ (Since Flow is steady)

From the momentum diagram,

$(Note\ that\ v_1 = v_2\ by\ using\ Bernoulli's\ eqn.)$

$$\sum F_X = -F = \sum_{CS} (\dot{m}v)_{outX} - \sum_{CS} (\dot{m}v)_{inX} = -\dot{m}v\cos B - \dot{m}v = -\dot{m}v(\cos B + 1)$$

$$F = \rho A v^2 (1 + \cos B)$$

END OF LECTURE (3)